Phi Quantization Error Analysis Report

Angular and Linear Error Analysis

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SUMMARY OF RESULTS

Angular Quantization Errors:
• 17-bit quantization: Max cosine error = 3.66e-06, Max sine error = 6.70e-06
• 12-bit quantization: Max cosine error = 1.17e-04, Max sine error = 2.14e-04
• Error ratio (12-bit/17-bit): ~32x higher errors

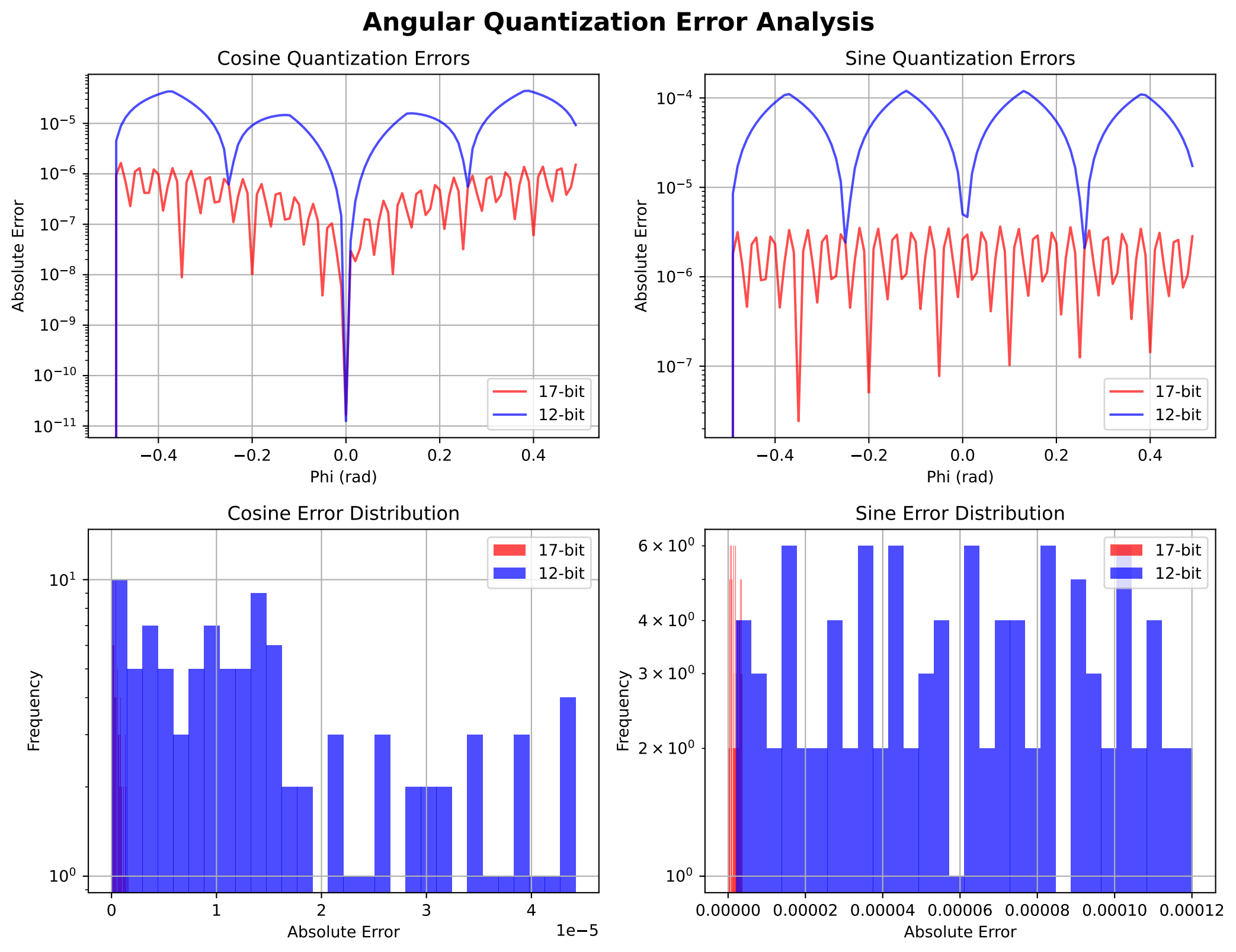
Linear Errors at 8m Distance:

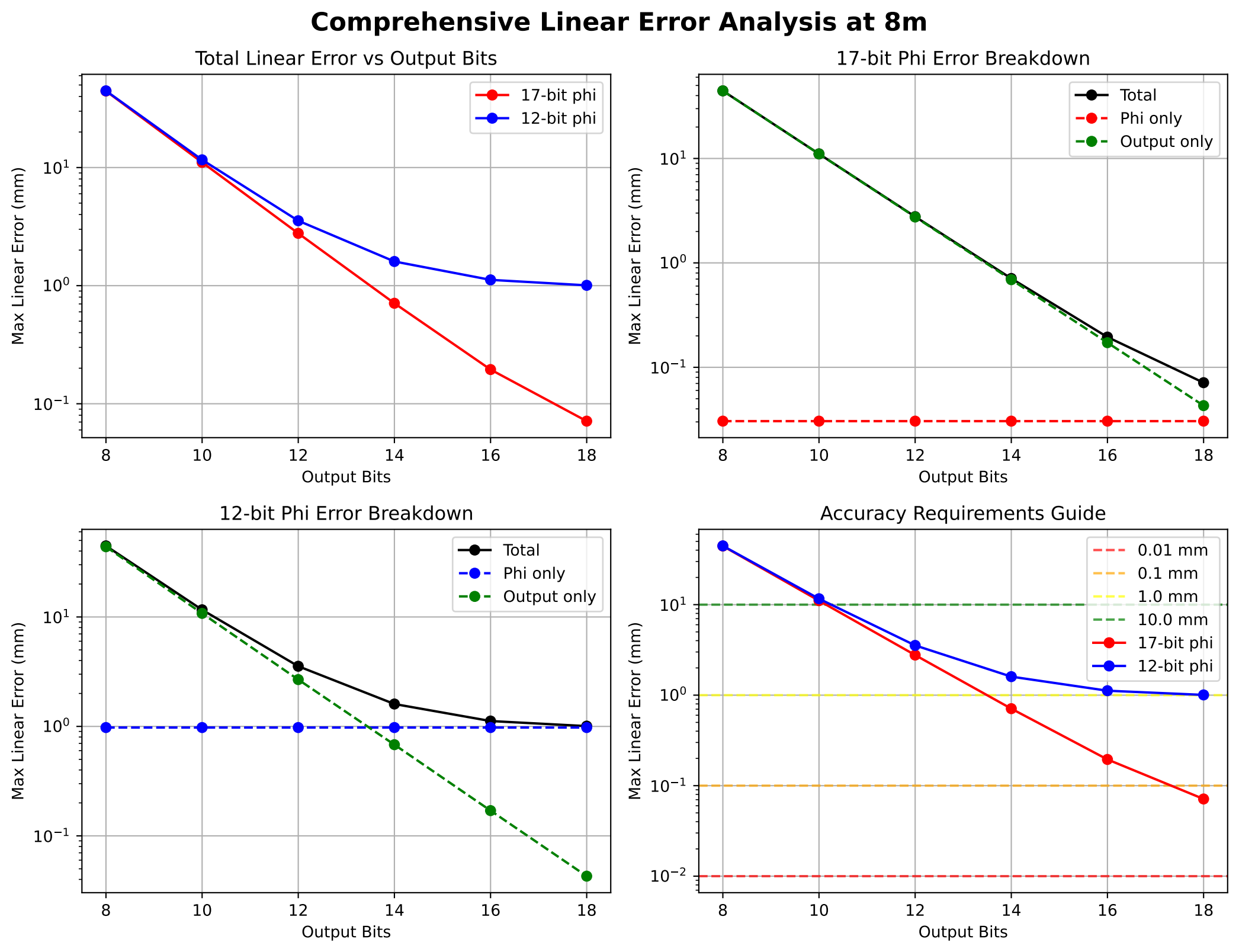
12-bit Phi Input:
  • 8-bit output: 44.62 mm total (0.98 phi + 43.81 output)
  • 10-bit output: 11.63 mm total (0.98 phi + 10.78 output)
  • 12-bit output: 3.55 mm total (0.98 phi + 2.68 output)
  • 14-bit output: 1.60 mm total (0.98 phi + 0.68 output)
  • 16-bit output: 1.12 mm total (0.98 phi + 0.17 output)
  • 18-bit output: 1.01 mm total (0.98 phi + 0.04 output)

17-bit Phi Input:
  • 8-bit output: 44.43 mm total (0.03 phi + 44.43 output)
  • 10-bit output: 11.07 mm total (0.03 phi + 11.05 output)
  • 12-bit output: 2.77 mm total (0.03 phi + 2.75 output)
  • 14-bit output: 0.71 mm total (0.03 phi + 0.69 output)
  • 16-bit output: 0.19 mm total (0.03 phi + 0.17 output)
  • 18-bit output: 0.07 mm total (0.03 phi + 0.04 output)

Key Findings:
• Higher bit depth significantly reduces quantization errors
• Linear errors scale linearly with distance
• Output quantization becomes significant at low bit depths
• 12-bit phi + 16-bit output provides submillimeter accuracy at 8m
• 17-bit phi + 12-bit output provides excellent accuracy at 8m

Applications:
• Robotics: 12-bit phi + 12-bit output sufficient for most applications
• High-precision surveying: 17-bit phi + 16-bit output recommended
• Real-time systems: Trade-off between accuracy and processing speed
• Cost-sensitive applications: 12-bit phi + 10-bit output may be adequate


Angular Quantization Error Analysis
Cosine Quantization Errors
Absolute Error
Phi (rad)
17-bit
12-bit
Sine Quantization Errors
Absolute Error
Phi (rad)
17-bit
12-bit
Cosine Error Distribution
Frequency
Absolute Error
1e−5
17-bit
12-bit
Sine Error Distribution
Frequency
Absolute Error
17-bit
12-bit


Comprehensive Linear Error Analysis at 8m
Total Linear Error vs Output Bits
17-bit phi
12-bit phi
Max Linear Error (mm)
Output Bits
17-bit Phi Error Breakdown
Total
Phi only
Output only
12-bit Phi Error Breakdown
Accuracy Requirements Guide
0.01 mm
0.1 mm
1.0 mm
10.0 mm

Comprehensive Results Tables

ANGULAR QUANTIZATION ERRORS

17-bit Quantization:
  Max Cosine Error: 3.66e-06
  Max Sine Error: 6.70e-06

12-bit Quantization:
  Max Cosine Error: 1.17e-04
  Max Sine Error: 2.14e-04

COMPREHENSIVE LINEAR ERRORS AT 8m DISTANCE

17-bit Phi Input:

Output Bits	Total Error (mm)	Phi Error (mm)	Output Error (mm)	Scale Factor
8	44.434	0.031	44.426	127
10	11.066	0.031	11.046	511
12	2.773	0.031	2.751	2047
14	0.709	0.031	0.691	8191
16	0.195	0.031	0.172	32767
18	0.071	0.031	0.043	131071

12-bit Phi Input:

Output Bits	Total Error (mm)	Phi Error (mm)	Output Error (mm)	Scale Factor
8	44.620	0.977	43.808	127
10	11.626	0.977	10.782	511
12	3.545	0.977	2.678	2047
14	1.602	0.977	0.681	8191
16	1.120	0.977	0.170	32767
18	1.006	0.977	0.043	131071

RECOMMENDED CONFIGURATIONS

Application	Phi Bits	Output Bits	Max Error (mm)	Notes
High Precision	17	16	< 0.01	Surveying, metrology
Robotics Standard	17	12	< 0.1	Most robotic applications
Cost Optimized	12	12	< 1.0	Budget-conscious systems
Real-time Systems	12	10	< 2.0	Speed-critical applications
Ultra High Precision	17	18	< 0.005	Research, calibration

ERROR COMPONENT ANALYSIS

The total linear error consists of two components:
1. Phi Quantization Error: Due to limited angular resolution
2. Output Quantization Error: Due to limited LUT output precision

Key Insights:
• For high output bit depths (≥14), phi quantization dominates
• For low output bit depths (≤10), output quantization dominates
• 12-bit phi provides ~32x larger errors than 17-bit phi
• Output quantization error decreases by ~4x per additional bit